PHYSICAL REVIEW E VOLUME 60, NUMBER 2 AUGUST 1999

Suppression of transverse instabilities for vector solitons

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(Received 8 February 1999; revised manuscript received 5 May 1999)

We analyze the transverse instability of *two-component spatial solitons* in a saturable nonlinear medium, in relation to recent experimental observations of spatial vector solitons in photorefractive media. We present the stability analysis for all three realizations: dark-bright, bright-bright, and dark-dark soliton pairs, and demonstrate that both the nonlinearity saturation and incoherent mode interaction can lead to *a strong suppression* of the soliton transverse instabilities. [S1063-651X(99)51708-X]

PACS number(s): 42.65.Tg, 05.45.Yv

Wave instabilities are probably the most dramatic physical effects that occur in nonlinear systems. They can lead not only to the beam filamentation and self-focusing, but also to a decay of solitary waves due to the symmetry-breaking perturbations at higher dimensions. Different types of nonlinearity-induced instabilities are known, such as modulational instability, self-focusing instability, transverse instability of planar solitons, etc. It is commonly believed that the instabilities are *strongly enhanced for coupled waves*, that is, when two (or more) waves coexist and interact. For example, this is observed for modulational instability, where the cross-phase modulation can even generate an instability of otherwise stable waves [1], for nonlinear focusing, where for two copropagating waves the nonlinear coupling reduces the critical power at least by a factor of 3 (see, e.g., Ref. [2]), or it causes waves which do not focus by themselves to focus because of their collective interaction, and so on.

Following the discovery of photorefractive spatial solitons, *vector solitons* were suggested to exist in photorefractive media, in several forms. One of these forms is of a particular interest, because it applies to any noninstantaneous nonlinearity and allows more than two components: vector solitons based on *mutual incoherence* between the various constituents [3]. Experimental observations of such solitons in three realizations: bright-bright, dark-dark, and dark-bright coupled pairs, have been recently reported [4]. However, unlike all earlier experiments with temporal vector solitons in fibers [5] and with spatial vector solitons in slab waveguides [6], the soliton pairs observed in Ref. [4] were generated in a *bulk* saturable nonlinear medium. This observation is in sharp contrast with the early belief that the transverse instability necessarily leads to a decay of the soliton stripe in a bulk. Furthermore, a very recent paper has reported the observation of multimode vector solitons [7], also employing the photorefractive saturable nonlinearity in a 3D bulk medium.

In this Rapid Communication, being *inspired by the recent experimental observation of stable incoherently coupled soliton pairs*, we investigate the transverse instability of all types of two-component vector solitons. We find that for saturable media, it is the nonlinearity saturation which leads to the suppression of the transverse instability. The most intriguing result is for the dark-bright vector pair, for which *even in the absence of saturation*, i.e., Manakov limit, the nonlinear mode coupling leads to a strong transverse stabilization. In other words, the growth rate of the snakelike transverse instability of a single dark soliton in bulk Kerr media is dramatically reduced *solely* due to the presence of a bright component, in sharp contrast to what was believed before. Moreover, the suppression of the transverse instability is enhanced when the amplitude of the bright component grows.

We start from the normalized equations [3]

$$i\frac{\partial U}{\partial z}+\frac{1}{2}\nabla_{\perp}^{2}U-\frac{\gamma U}{1+|U|^{2}+|V|^{2}}=0,$$

$$i\frac{\partial V}{\partial z}+\frac{1}{2}\nabla_{\perp}^{2}V-\frac{\gamma V}{1+|U|^{2}+|V|^{2}}=0, \qquad (1)$$

where U,V are the envelopes of the two interacting beams, $\gamma=\beta(1+\rho)$, where ρ is the total intensity at infinity and β is the peak nonlinear index change, and $\nabla_{\perp}^{2}=\partial^{2}/\partial x^{2}+\partial^{2}/\partial y^{2}$ is the transverse Laplacian. Equations (1) describe two coupled beams in a saturable optical medium with a refractive index change proportional to $1/(1+|U|^{2}+|V|^{2})$. Such an interaction can form *vector solitons* that consist of two (or more) components mutually self-trapped in a nonlinear medium. In the small-intensity (Kerr) limit, the governing equations (1) describe the so-called *Manakov solitons* [6,8].

Because it is well established that both bright and dark scalar Kerr solitons (solutions of a single cubic NLS equation) are unstable to the symmetry-breaking instabilities of higher dimensions [9,10], the commonly held belief is that vector solitons are not observable in higher dimensions either and, moreover, they should be even more unstable due to the mode interaction. Here we resolve this question by analyzing the transverse instability of all types of vector soliton pairs [11]. The dark-bright soliton pair is the most interesting case from the physics standpoint and nontrivial for the analysis. It turns out that the effect of the mutual interaction between the soliton components of *different* types has never been addressed for any kind of instability analysis. Here we consider first the transverse instability of the dark-bright soliton pairs.

Dark-bright soliton pairs. We look for the stationary solutions in the form of bright, $U_0(z,x,y)=u(x)e^{i\mu z}$, and dark, $V_0(z,x,y)=v(x)e^{i\nu z}$, components defined by the boundary conditions $u(\pm\infty)\to 0$ and $v(\pm\infty)\to\pm\sqrt{\rho}$, respectively. Equations of motion for the normalized envelopes u and v are

$$\frac{1}{2}\frac{d^2u}{dx^2}-\left(\mu+\frac{\gamma}{1+u^2+v^2}\right)u=0,$$

$$\frac{1}{2}\frac{d^2v}{dx^2}-\left(\nu+\frac{\gamma}{1+u^2+v^2}\right)v=0. \qquad (2)$$

Analyzing the asymptotics of $v(x)$ as $x\to+\infty$, one can show that a dark component exists only as $\nu=-\beta>0$ along with a nontrivial bright component $u(x)$, as long as $u(0)<\sqrt{\rho}$ [3]. The other option, for $u(0)>\sqrt{\rho}$ and $\beta>0$, is modulationally unstable [3,4]. Though the analysis is done below for the nonlinearity (1), our theory of transverse instability of coupled dark-bright solitons is general and holds for any kind of nonlinearity supporting such soliton pairs.

Let us write weakly perturbed solutions of Eqs. (1) in the form $U(z,x,y)=[u(x)+\epsilon U_1(z,x,y)]e^{i\mu z}$, $V(z,x,y)=[v(x)+\epsilon V_1(z,x,y)]e^{i\nu z}$, where each perturbation is expressed as a superposition of plane waves with the wave number q and frequency ω:

$$U_1=\phi_1(x)e^{i\omega z+iqy}+\phi_2^*(x)e^{-i\omega^* z-iqy},$$

$$V_1=\psi_1(x)e^{i\omega z+iqy}+\psi_2^*(x)e^{-i\omega^* z-iqy}.$$

By writing $\phi_{1,2}=(\phi^+\pm\phi^-)/2$, $\psi_{1,2}=(\psi^+\pm\psi^-)/2$ we arrive, at the first order in ϵ, at the following linear eigenvalue problem:

$$\frac{1}{2}\frac{d^2\phi^+}{dx^2}-\left(\mathcal{V}_\mu(x)+\frac{1}{2}q^2\right)\phi^+-\omega\phi^-+\mathcal{U}(x)\psi^+=0,$$

$$\frac{1}{2}\frac{d^2\psi^+}{dx^2}-\left(\mathcal{V}_\nu(x)+\frac{1}{2}q^2\right)\psi^+-\omega\psi^-+\mathcal{U}(x)\phi^+=0,$$

$$\frac{1}{2}\frac{d^2\phi^-}{dx^2}-\left(\mu+\frac{1}{2}q^2+\frac{\gamma}{1+u^2+v^2}\right)\phi^--\omega\phi^+=0,$$

$$\frac{1}{2}\frac{d^2\psi^-}{dx^2}-\left(\nu+\frac{1}{2}q^2+\frac{\gamma}{1+u^2+v^2}\right)\psi^--\omega\psi^+=0,$$

where $\mathcal{V}_\mu(x)$, $\mathcal{V}_\nu(x)$, and $\mathcal{U}(x)$ are defined as

$$\mathcal{V}_\mu(x)=\mu+\frac{\gamma(1-u^2+v^2)}{(1+u^2+v^2)^2},$$

$$\mathcal{V}_\nu(x)=\nu+\frac{\gamma(1+u^2-v^2)}{(1+u^2+v^2)^2},\quad \mathcal{U}(x)=\frac{2\gamma uv}{(1+u^2+v^2)^2}.$$

It is impossible to solve the above spectral problem exactly, i.e., to calculate the eigenvalue spectrum $\omega(q)$. Therefore, we restrict ourselves to the long-wave limit when the soliton size is assumed small in comparison with the perturbation scale. This means that the solution of the above system may be found in the asymptotic form $\phi^\pm\simeq\phi_0^\pm+q\phi_1^\pm+q^2\phi_2^\pm+\cdots$, $\psi^\pm\simeq\psi_0^\pm+q\psi_1^\pm+q^2\psi_2^\pm+\cdots$, $\omega(q)=q\omega_1+q^2\omega_2+\cdots$, leading to the following sets of equations:

$$O(q^0):\quad \mathcal{J}\boldsymbol{m}_0^+=0,\quad \mathcal{J}_\mu\phi_0^-=0,\quad \mathcal{J}_\nu\psi_0^-=0;$$

$$O(q^1):\quad \mathcal{J}\boldsymbol{m}_1^+=\omega_1\boldsymbol{m}_0^-,\quad \mathcal{J}_\mu\phi_1^-=\omega_1\phi_0^+,$$

$$\mathcal{J}_\nu\psi_1^-=\omega_1\psi_0^+;$$

$$O(q^2):\quad \mathcal{J}\boldsymbol{m}_2^+=\omega_1\boldsymbol{m}_1^-+\omega_2\boldsymbol{m}_0^-+1/2\boldsymbol{m}_0^+,$$

$$\mathcal{J}_\mu\phi_2^-=\omega_1\phi_1^++\omega_2\phi_0^++1/2\phi_0^-,$$

$$\mathcal{J}_\nu\psi_2^-=\omega_1\psi_1^++\omega_2\psi_0^++1/2\psi_0^-,$$

where $\boldsymbol{m}^\pm=(\phi^\pm,\psi^\pm)$ and the operators $\mathcal{J}$, $\mathcal{J}_\mu$, and $\mathcal{J}_\nu$ are defined by

$$\mathcal{J}=\begin{pmatrix}\frac{1}{2}\frac{d^2}{dx^2}-\mathcal{V}_\mu(x) & \mathcal{U}(x)\\ \mathcal{U}(x) & \frac{1}{2}\frac{d^2}{dx^2}-\mathcal{V}_\nu(x)\end{pmatrix},$$

$$\mathcal{J}_s=\frac{1}{2}\frac{d^2}{dx^2}-\left(s+\frac{\gamma}{1+u^2+v^2}\right),$$

where $s=\mu,\nu$. Keeping in mind that only localized perturbations can lead to instability, we attempt to construct, at each order in q, a localized solution to the above system. Scrutinizing the zero order equation shows that ϕ_0^-, ψ_0^-, and $\boldsymbol{m}_0^+$ are the neutral modes $(\omega=q=0)$, corresponding to the gauge transformation and translational symmetry of Eqs. (1), respectively. Hence, $\phi_0^-(x)=c_1u(x)$, $\psi_0^-(x)=c_3v(x)$, $\phi_0^+(x)=c_2u_x(x)$, and $\psi_0^+(x)=c_2v_x(x)$, where the index x stands for the corresponding derivative. All modes except ψ_0^- are localized and belong to bound states. By means of the multiscale expansion technique, we can show that the mode with $c_3\neq 0$ does not lead to instability. Therefore, for simplicity, we set $c_3=0$. Then, the solution to the first order is given by $\phi_1^+(x)=c_1\omega_1(du/d\mu)$, $\psi_1^+(x)=c_1\omega_1(dv/d\mu)$, $\phi_1^-(x)=c_2\omega_1xu(x)/2$, $\psi_1^-=c_2\omega_1[xv(x)-\rho v(x)\Gamma(x)]/2$, where $\Gamma(x)$ is defined by the relation $d\Gamma(x)/dx=1/v^2(x)$.

Analyzing those results, we find *two instability modes*. First, for $c_1\neq 0$, ω_1 is obtained from the solvability condition for ϕ_2^- in the second order in q, i.e., orthogonality of an inhomogeneous part to the eigenfunction $u(x)$. In this way, we retrieve the scalar result of [9], where a long-wave analytical expansion for the instability growth rate, $i\omega=i\omega_1q+\cdots$ was derived. In our notation, this gives

$$\omega_1^2=-\frac{P}{(dP/d\mu)}, \qquad (3)$$

where P is the soliton power, $P=\int_{-\infty}^{+\infty}|u|^2dx$. From the multiscale analysis, we can show that the solvability condition for the function ψ_2^- is satisfied and that ψ_2^- is indeed a

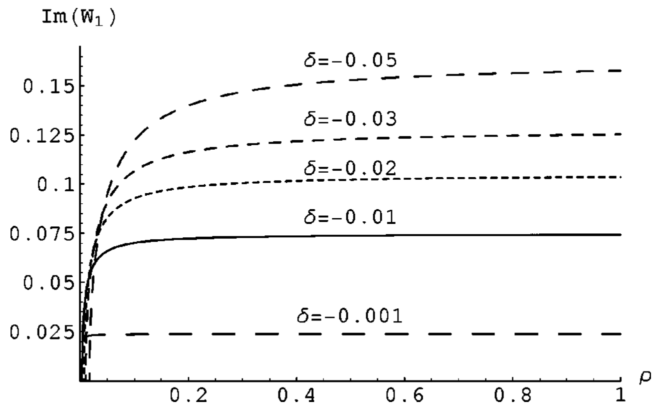

FIG. 1. Growth rate of the transverse instability of a dark-bright soliton pair for different values of δ.

bounded function. The second instability mode follows from the solvability conditions for $\boldsymbol{m}_2^+$, and its frequency can be written in the form

$$\omega_1^2=\frac{\int_{-\infty}^{+\infty}[u_x^2(x)+v_x^2(x)]dx}{\mathcal{P}+2\rho\lim_{x\to\infty}[v^2(x)\Gamma(x)-x]},\tag{4}$$

where $\mathcal{P}$ is the complementary power defined as $\mathcal{P}=\int_{-\infty}^{+\infty}dx[u^2(x)+v^2(x)-\rho]$. By considering the asymptotics at $x\to+\infty$, we can show that the value $\lim_{x\to\infty}[v^2(x)\Gamma(x)-x]$ is always finite. We emphasize that the new condition (4) derived above is general, and it holds for any kind of nonlinearity that supports dark-bright soliton pairs. Moreover, in the case of a single dark soliton ($u=0$, $v=\tanh x$) from Eq. (4) we recover immediately the result of Ref. [10]. In a particular case discussed in Ref. [3], we can use an approximate analytic solution, $u(x)=\sqrt{r}\,\mathrm{sech}[(\beta\delta)^{1/2}x]$ and $v(x)=\sqrt{\rho}\tanh[(\beta\delta)^{1/2}x]$, where $\delta\equiv(r-\rho)/(1+\rho)<0$; $|\delta|\ll1$ and the propagation constants are $\mu\simeq-\beta(1-\delta/2)$ and $\nu=-\beta$. In this case, the condition (3) does not lead to instability because $dP/d\mu<0$. This is in contradistinction with the bright solitons and bright-bright pairs, for which $\beta>0$; here $\beta<0$, i.e., a self-defocusing nonlinearity. However, applying the second condition (4), we obtain the instability growth rate $\omega_1^2=\beta\delta(r+2\rho)/3(r-2\rho)$. From the experimental parameters of Ref. [4], we take $\beta\simeq0.566$, and show this result in Fig. 1 for different values of δ. Another important example is the dark-bright Manakov solitons [12]. In this case, the bright and dark components are given by the expressions [12]: $u(x)=\sqrt{1-a^2}\,\mathrm{sech}(ax)$ and $v(x)=\tanh(ax)$ with the propagation constant $\mu=-(1-a^2/2)$, where a ($a^2<1$) characterizes the amplitude of the bright component for the normalized background. Again, the condition (3) does not lead to any instability because $dP/d\mu<0$. However, from Eq. (4) we obtain

$$\omega_1^2=\frac{a^2(a^2-3)}{3(a^2+1)}.\tag{5}$$

When $a=1$, i.e., for a single dark soliton, we retrieve the result of [10], $\omega_1^2=-1/3$. In general, the result (5) reveals *an unexpected feature* of the dark-bright soliton pairs: *a bright component, embedded in a defocusing medium, leads to an effective suppression of the transverse instability of a dark soliton*. This instability suppression results *only* from the presence of the bright component and becomes stronger as we increase the bright-soliton intensity.

Bright vector solitons. Next, for bright vector solitons, or bright-bright soliton pairs, a solution of Eq. (1) that corresponds to vanishing boundary conditions and $\rho=0$, can be obtained by the substitution $U_0(z,x,y)=\sqrt{r}\cos\theta Y(x)e^{i\mu z}$, $V_0(z,x,y)=\sqrt{r}\sin\theta Y(x)e^{i\mu z}$, where r is the total peak intensity, $Y(x)$ is a normalized (real) amplitude, θ is an arbitrary angle, and μ is a propagation constant. We consider a fundamental soliton stripe described by the same shape $Y(x)$ for both U_0 and V_0 and uniform in the direction of applied perturbation, y, where $Y(x)$ satisfies the following scalar equation:

$$\frac{d^2Y}{dx^2}-2\mu Y-\frac{2\beta Y}{1+rY^2}=0,\tag{6}$$

and it describes a one-parameter family of bright solitons for $\beta>0$ and $\mu=-(\beta/r)\ln(1+r)$ [13,14]. Now, we consider a steady-state soliton solution perturbed by a small perturbation. Notice that Eqs. (1) are invariant with respect to the transformation $U_0'=U_0\cos\alpha+V_0\sin\alpha$, $V_0'=-U_0\sin\alpha+V_0\cos\alpha$, which allows one to map the problem back to the scalar case with the solutions $U_0'=\sqrt{r}Y(x)e^{i\mu z}\equiv u(x)e^{i\mu z}$, $V_0'=0$. This simplifies the corresponding eigenvalue problem. Indeed, if we write a perturbed solution in the form (omitting primes) $U(z,x,y)=[u(x)+\epsilon U_1(z,x,y)]e^{i\mu z}$, and $V(z,x,y)=\epsilon V_1(z,x,y)e^{i\mu z}$, where $\epsilon\ll1$, then from Eqs. (1) we can obtain *a system of two decoupled equations*,

$$\left(i\frac{\partial}{\partial z}+\frac{1}{2}\nabla_\perp^2-\mu\right)U_1-\frac{\gamma(U_1-u^2U_1^*)}{(1+u^2)^2}=0,\tag{7}$$

$$i\frac{\partial V_1}{\partial z}+\frac{1}{2}\nabla_\perp^2V_1-\left(\mu+\frac{\gamma}{1+u^2}\right)V_1=0.\tag{8}$$

Importantly, Eq. (7) is a linearized nonlinear Schrödinger (NLS) equation for a saturable nonlinear medium, and its stability analysis follows from the theory developed for a scalar NLS equation [9]. Applying the result (3) to a saturable nonlinearity, we find numerically the dependence of the growth rate ω_1 on the beam peak amplitude $u_0\equiv\sqrt{r}$ as shown in Fig. 2. Thus, the growth rate of the soliton transverse instability is decreasing with an increasing ratio between the peak soliton intensity and the saturation intensity, u_0^2. In the low-intensity (Kerr) limit of the saturable nonlinearity, a strong transverse instability takes place, and it is typical of *scalar* Kerr solitons. The comparison with the experiments of Ref. [4] reveals the same trend: the instability is strongly suppressed with increasing saturation, and that is why such bright-bright soliton pairs were observed experimentally in the saturated regime only. On the other hand, in the low-intensity limit the strong transverse instability did not make it possible to observe coupled bright soliton pairs [4].

Next, we investigate the second equation (8). Let $V_1(z,x,y)=G(x)e^{i\omega z+iqy}$, where q is the perturbation wave

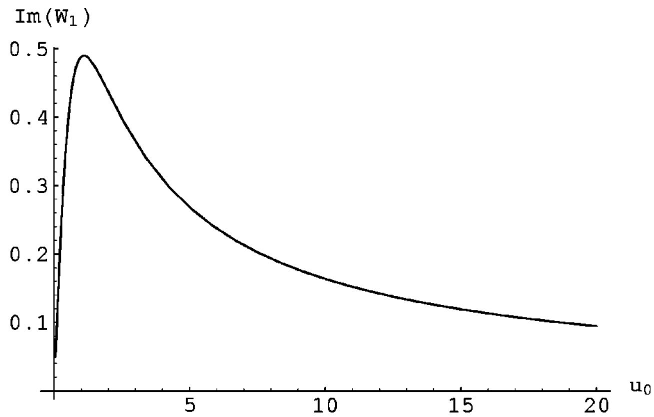

FIG. 2. Growth rate of the soliton long-wavelength transverse instability vs saturation intensity.

number and ω is the corresponding growth rate. Substituting this into Eq. (8), we obtain the eigenvalue equation

$$\frac{1}{2}\frac{d^2G}{dx^2}-\left(\mu+\frac{\gamma}{1+u^2}+\frac{1}{2}q^2\right)G=\omega G, \tag{9}$$

with the boundary conditions $G(\pm\infty)\rightarrow 0$. It is easy to verify that Eq. (9) is a self-adjoint eigenvalue problem with a real spectrum $\omega(q)$. This implies that the transverse instability of bright-bright vector solitons is completely defined by the corresponding scalar problem.

Dark vector solitons. Dark solitons are two-component kinklike solutions of Eqs. (1) with antisymmetric field profile and nonvanishing asymptotics. For this case, $\rho\neq 0$ and a solution to Eqs. (1) can be sought in the form $U_0(z,x,y)=\sqrt{\rho}Y(x)e^{i\nu z}\equiv v(x)e^{i\nu z}, V_0=0$, where we have used the rotational invariance of Eqs. (1). Function $v(x)$ is a solution of Eq. (6) with the boundary conditions $v(\pm\infty)\rightarrow\pm\sqrt{\rho}$ which leads to the condition $\nu=-\beta>0$. Linearizing Eqs. (1) around this solution, we obtain the same linear system (7) and (8). The eigenvalue equation (7) has been recently analyzed in Ref. [15], and it has been revealed that the nonlinearity saturation leads to a strong suppression of the soliton transverse instability (see Fig. 1 of Ref. [15]). What remains to be checked for our vector soliton case is the effect of the other mode perturbation described by Eq. (8) on the soliton stability. Following the same reasoning as above, we find that at least in the long-wave limit, Eq. (9) has no solutions corresponding to discrete unstable eigenmodes. This implies that the case of dark vector solitons is also reduced to a scalar problem.

In conclusion, we have analyzed the transverse instability of three possible realizations of two-component vector solitons in a bulk nonlinear medium. In the case of dark-bright soliton pairs, we have derived, in the long-wave approximation, a general result for the instability growth rate, and demonstrated that the incoherent mode interaction can lead to a strong suppression of the soliton transverse instabilities. In the case of rotationally invariant nonlinearity, the cases of bright and dark vector solitons are shown to map to the corresponding scalar problem.

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